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Generalised Estimation Under Successive Sampling

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Summary

Paper presents generalised estimation under successive sampling. The estimation of mean for recent occasion, change over occasion and average of several occasions are particular cases of the proposed generalised estimation. The usefulness of the results are illustrated with the help of numerical illustration.

Key words : Successive Sampling, Replacement fraction, Minimum Variance Estimator.

Introduction

The theory of successive sampling for dynamic population was developed by Jessen [2], Yates [8], Patterson [3], Eckler [1], Rao & Grahm [5] and others. In a recent paper by Narain et al. [4] different aspects of estimation of change in agricultural surveys have been examined including the change in the sampling structure of population as well as the change in the parameters under study.

In such surveys, one may be interested in any of the following aspects.

i) The population parameters for the most recent period.

ii) Change taking place in the population parameters.

iii) The population parameters for the entire period under consideration.

Many times it may be of interest to estimate linear function of means of several occasions e.g. the average real income after adjustment for prices, the number of births averted by family planning methods (the converter may be different for different methods and for the same method over time). The estimation of mean for the current occasion, change over occasion, average over all occasions can be shown as particular case of estimation of linear function of the average of occasions. For two occasions let us consider a weighted function of means of two occasions as

$$A\,\overline{X} + B\,\overline{Y}$$

where $\overline{X}$ = Mean of previous occasion.

$\overline{Y}$ = Mean of current occasion.

A & B are weights.

If A = 0, B = 1	then above expression reduces to $\overline{Y}$ i.e. mean of the current occasion.
If A = -1, B = 1,	the above expression reduces to the change of means over two occasions.
If A = B = 1/2,	then it reduces to the average of means over two occasions.

It is, therefore, of interest to develop an appropriate estimation procedure in such situations. The present paper is an attempt in this direction. It deals with estimation of linear function of means for two occasions.

Notation and Estimation :

Consider a simple situation of sampling over two occasions. Let

n =	Sample Size Selected by SRSWOR from N on first occasion.
m =	(np) Sub Sample Size for second occasion drawn from remaining N-n units by SRSWOR.
y =	Value of character on the recent occasion.
x =	Value for character in previous occasion.
p =	Replacement fraction so that p + q = 1
$\overline{y}^1$=	Mean of character on the matched unit for the current occasion.

$\bar{y}^{11}$ = Mean of 2nd occasion on unmatched unit for the current occasion.

$\bar{x}^{1}$ = Mean of character on matched unit for the previous occasion.

$\bar{x}^{1}$ = Mean of character on unmatched sample from previous occasions.

Consider the parameter of interest :

$$\theta = A\bar{X} + B\bar{Y}$$

Consider the estimator :

$$\bar{y} = a\bar{x}^{1} + b\bar{x}^{11} + c\bar{y}^{1} + d\bar{y}^{11}$$

The values of a, b, c and d can be worked out in such a way that

$$E(\bar{y}) = A\bar{X} + B\bar{Y}$$

In this case

$$b = A - a$$

and $d = B - c$

The estimator then reduces to

$$\bar{y} = a\bar{x}^{1} + (A-a)\bar{x}^{11} + c\bar{y}^{1} + (B-c)\bar{y}^{11}$$

The variance of the estimator is given by

$$V(\bar{y}) = a^2 V(\bar{x}^{1}) + (A-a)^2 V(\bar{x}^{11}) + c^2 V(\bar{y}^{1}) + (B-c)^2 V(\bar{y}^{11})$$
$$+ 2a(A-a)\,Cov\,(\bar{x}^{1}\ \ \bar{x}^{11}) + 2ac\,Cov\,(\bar{x}^{1}\ \ \bar{y}^{1})$$
$$+ 2c\,(A-a)\ \ Cov\,(\bar{x}^{11}\ \ \bar{y}^{1})$$

If $\bar{y}$ is a minimum variance unbiased estimator, it must be uncorrelated with every zero functions (Theorem 1.10 due to Rao [5])

Thus, in usual notations

$$V(\bar{y}) = a^2 \frac{Sy^2}{m} + (A-a)^2 \frac{Sy^2}{n-m} + c^2 \frac{Sy^2}{m}$$

$$+ (B-c)^2 \frac{Sy^2}{n-m} + 2ac\,\rho \frac{Sy^2}{m}$$

$$V(\bar{y}) = a^2 \frac{Sy^2}{m} + (A^2 + a^2 - 2Aa) \frac{Sy^2}{n-m} + c^2 \frac{Sy^2}{m}$$

$$+ (B^2 + c^2 - 2Bc) \frac{Sy^2}{n-m} + 2ac\,\rho \frac{Sy^2}{m}$$

$$V(\bar{y}) = a^2 \left(\frac{Sy^2}{m} + \frac{Sy^2}{n-m}\right) + (A^2 - 2Aa) \frac{Sy^2}{n-m}$$

$$+ c^2 \left(\frac{Sy^2}{m} + \frac{Sy^2}{n-m}\right) + (B^2 - 2Bc) \frac{Sy^2}{n-m} + 2ac\,\rho \frac{Sy^2}{m}$$

The values of a & c are calculated by differentiating the above function with respect to a & c and equating to zero i.e.

$$2a \left(\frac{Sy^2}{m} + \frac{Sy^2}{n-m}\right) - 2A \frac{Sy^2}{n-m} + 2c\,\rho \frac{Sy^2}{m} = 0$$

$$an - Am + c\,\rho\,(n-m) = 0$$

$$a = \frac{-c\,\rho\,(n-m) + Am}{n}$$

$$2c \left(\frac{Sy^2}{m} + \frac{Sy^2}{n-m}\right) - 2B \frac{Sy^2}{n-m} + 2a\,\rho \frac{Sy^2}{m} = 0$$

$$cn - mB + a\,\rho\,(n-m) = 0$$

putting value of a and solving these two equations we get,

$$C = \frac{p(B - A\,\rho\,q)}{1 - \rho^2 q^2}$$

and $$a = -\frac{p\,(B - A\,\rho q)\,\rho\,q}{1 - \rho^2 q^2} + Ap$$

Substituting the values of a & c in variance expression, variance expression simplifies as :

$$V(\bar{y}) = \frac{Sy^2}{(1-\rho^2 q^2)} \frac{1}{n} [A^2 + B^2 + B^2\rho^2 pq + A^2 \rho^2 pq - 2a^2 \rho^2 q$$
$$- 2B^2\rho^2 q + A^2 \rho^4 q^3$$
$$+ B^2\rho^4 q^3 - 2AB\, p\, q^2 \rho^3$$
$$+ 2AB\, p\rho]$$

It may be seen that for A = 0 and B = 1, the optimum value of a & c and V are given by

$$a = \frac{-pq\rho}{1-\rho^2 q^2}, \quad c = \frac{p}{1-\rho^2 q^2}$$

$$V(\bar{y}) = \frac{Sy^2}{n} \frac{1-\rho^2 q}{1-\rho^2 q^2}$$

This corresponds to the results for estimate of mean for current occasion.

Similarly for A = 1 & B = -1, the values of a, c & v are obtained as

$$a = \frac{p(1+q\rho)}{1-\rho^2 q^2}, \quad c = \frac{-p(1+q\rho)}{1-\rho^2 q^2}$$

$$V = \frac{2Sy^2}{n} \frac{1-\rho}{1-q\rho}$$

This corresponds to estimate of change in the means of two occasions. Further for A = B = 1/2, the values of a, c and v are given by

$$a = \frac{p(\tfrac{1}{2} - \tfrac{1}{2} q\rho)}{1-\rho^2 q^2}, \quad c = \frac{-p(\tfrac{1}{2} - \tfrac{1}{2} q\rho)}{1-\rho^2 q^2}$$

$$V = \frac{Sy^2}{2n} \frac{1+\rho}{1+q\rho}$$

This corresponds to estimate of average of means over two occasions.

Thus, the results obtained are the generalised one and the known results for estimate of mean of current occasion, change in means over occasion and average of means for two occasions are particular cases of the generalised estimation.

Optimum Value of q

For obtaining the optimum value of q, differentiate the above function w.r.t. q and equate to Zero. Then we get

$$\frac{Sy^2}{n}\Big[(1-\rho^2q^2)^2(-B^2\rho^2-2B^2\rho^2q-A^2\rho^2-2A^2q\rho^2+3A^2q^2\rho^4$$
$$+3B^2q^2\rho^4-4ABq\rho^3+6ABq^2\rho^3-2AB\rho)$$
$$-2(1-\rho^2q^2)(-2\rho^2q)(A^2+B^2-B^2\rho^2q-B^2\rho^2q^2)$$
$$-A^2q\rho^2-A^2q^2\rho^2+A^2q^3\rho^4+B^2\rho^4q^3-2ABq^2\rho^3$$
$$+2ABq^3\rho^3+2AB\rho-2ABq\rho-2ABq\rho)\Big]/(1-\rho^2q^2)^4$$
$$=0$$

Above equation is a polynomial in q and the solution is not straight forward. The values of optimum q have been calculated empirically.

Empirical Illustration

The values of the variance of the estimator have been calculated for several situations (i.e. for various values of A & B). In each situation a wide range of correlation coefficient has been considered. The alternatives considered are the following :

Values of A & B			
Both A & B Positive		Either A or B Negative	
A	B	A	B
0.0	1.0	1.0	−1.0
1.0	0.0	−1.0	1.0
0.1	0.9	1.0	−0.5
0.9	0.1	−0.5	1.0
0.5	0.5	−0.1	1.0
0.3	0.7	−0.2	1.0

Contd....

Contd....

0.7	0.3	–0.3	1.0
1.0	0.1	–0.4	1.0
1.0	0.2	–0.5	1.0
1.0	0.3	–0.6	1.0
1.0	0.4	–0.7	1.0
1.0	0.5	–0.8	1.0
1.0	0.6	–0.9	1.0
1.0	0.7	1.0	–0.1
1.0	0.8	1.0	–0.2
1.0	0.9	1.0	–0.3
0.1	1.0	1.0	–0.4
0.2	1.0	1.0	–0.5
0.3	1.0	1.0	–0.6
0.4	1.0	1.0	–0.7
0.5	1.0	1.0	–0.8
0.6	1.0	1.0	–0.9
0.7	1.0		
0.8	1.0		
0.9	1.0		

The values of ρ considered are .6 to .9.

The results are as under:

For the values of A & B say 1 each, it is known that the optimum value of q is one. In cases A & B both take positive values like .7 & .3, .3 & .7, 1 & .2, 1 & .5 etc., the optimum value of q has been observed as near one. For the values of A & B say 1 & .1, 1 & .2, .2 & 1, .1 & 1 optimum value of q is less than one in the range .7 to .8 for various values of ρ in the range .7 to .9.

If one of values of A & B is 0 then optimum value of q is around .60 corresponding to the value of ρ in the range .6 to .8 and .70 corresponding to the value of ρ = .9. Similarly for the values of A & B as .9 & .1, optimum value of q is around .70 for in the range .6 to .8 and .8 for ρ = .9.

If A = 1, B = -1, it is known that that optimum value of q is zero.

For the values of A & B say 1 & .1, optimum value of q is .7 corresponding to the value of in the range .6 to .8 and optimum value of q is ..8 corresponding to the value ρ as .9.

For the values of A & B say 1 & .2, optimum value of q is .0 for ρ=.6 to .9 and for the values of A & B say 1 & .3 to .9, optimum value of q is one for ρ = .7 to .9.

For the values of A & B with one value + ve and another is small fraction with negative say B = 1, A = –.1, optimum value of q varies from .4 to .6 with ρ = .6 to .9. Similarly for the values of A & B say B = 1, A = -.2, optimum q decreases from .2 to .5 with ρ = .6 to .9, so optimum value of q approaches to zero as the value of B = 1, A= –.3, –.4, –.5 to –.9 with ρ = .6 to .9.

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Table. Optimum value of q for different values of A & B and ρ

A	B	ρ	Optimum q
0.0	1.0	0.7 0.8 0.9	0.6 0.6 0.7
1.0	0.0	0.7 0.8 0.9	0.6 0.6 0.7
0.1	0.9	0.6 0.7 0.8 0.9	0.7 0.7 0.7 0.8
0.9	0.1	0.6 0.7 0.8 0.9	0.7 0.7 0.7 0.8
0.5	0.5	0.7 0.8 0.9	1.0 1.0 1.0
0.3	0.7	0.7 0.8 0.9	1.0 1.0 1.0
0.7	0.3	0.7 0.8 0.9	1.0 1.0 1.0
0.1	1.0	0.6 0.7 0.8 0.9	0.7 0.7 0.7 0.8
0.2	1.0	0.6 0.7 0.8 0.9	0.8 0.8 0.8 0.8
0.3	1.0	0.7 0.8 0.9	1.0 1.0 1.0
0.4	1.0	0.7 0.8 0.9	1.0 1.0 1.0
0.5	1.0	0.7 0.8 0.9	1.0 1.0 1.0

A	B	ρ	Optimum q
0.6	1.0	0.7 0.8 0.9	1.0 1.0 1.0
0.7	1.0	0.7 0.8 0.9	1.0 1.0 1.0
0.8	1.0	0.7 0.8 0.9	1.0 1.0 1.0
0.9	1.0	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.1	0.7 0.8 0.9	0.7 0.7 0.8
1.0	0.2	0.7 0.8 0.9	0.8 0.8 0.8
1.0	0.3	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.4	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.5	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.6	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.7	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.8	0.7 0.8 0.9	1.0 1.0 1.0
1.0	0.9	0.7 0.8 0.9	1.0 1.0 1.0

A	B	ρ	Optimum q
1.0	−1.0	0.7 0.8 0.9	0.0 0.0 0.0
−1.0	1.0	0.7 0.8 0.9	0.0 0.0 0.0
−0.1	1.0	0.6 0.7 0.8 0.9	0.4 0.5 0.5 0.6
−0.2	1.0	0.6 0.7 0.8 0.9	0.2 0.3 0.4 0.5
−0.3	1.0	0.6 0.7 0.8 0.9	0.0 0.2 0.3 0.4
−0.4	1.0	0.7 0.8 0.9	0.0 0.2 0.3
−0.5	1.0	0.7 0.8 0.9	0.0 0.0 0.2
−0.6	1.0	0.7 0.8 0.9	0.0 0.0 0.0
−0.7	1.0	0.7 0.8 0.9	0.0 0.0 0.0
−0.8	1.0	0.7 0.8 0.9	0.0 0.0 0.0

A	B	ρ	Optimum q
−0.9	1.0	0.7 0.8 0.9	0.0 0.0 0.0
−0.1	1.0	0.7 0.8 0.9	0.0 0.0 0.0
1.0	−0.2	0.7 0.8 0.9	0.3 0.4 0.5
1.0	−0.3	0.7 0.8 0.9	0.2 0.3 0.4
1.0	−0.4	0.7 0.8 0.9	0.0 0.2 0.3
1.0	−0.5	0.7 0.8 0.9	0.0 0.0 0.2
1.0	−0.6	0.7 0.8 0.9	0.0 0.0 0.0
1.0	−0.7	0.7 0.8 0.9	0.0 0.0 0.0
1.0	−0.8	0.7 0.8 0.9	0.0 0.0 0.0
1.0	−0.9	0.7 0.8 0.9	0.0 0.0 0.0